11-7 Multiplying, Dividing, and Simplifying Radicals

Objective: To simplify products and quotients of radicals.

Vocabulary

Rationalize the denominator The process of eliminating a radical from the denominator of a fraction. Remember that $(\sqrt{a})^2 = a$.

Simplest form of a square-root radical

When all of the following are true:	Simplest form	*Not* in simplest form
1. No integral radicand has a perfect-square factor other than 1.	$2\sqrt{5}$	$\sqrt{20}$
2. No fractions are under a radical sign.	$\dfrac{\sqrt{3}}{3}$	$\sqrt{\dfrac{1}{3}}$
3. No radicals are in a denominator.	$\dfrac{5\sqrt{2}}{2}$	$\dfrac{5}{\sqrt{2}}$

Example 1 Simplify $2\sqrt{3} \cdot 3\sqrt{48}$.

Solution
$$2\sqrt{3} \cdot 3\sqrt{48} = (2 \cdot 3)(\sqrt{3} \cdot \sqrt{48})$$
$$= 6\sqrt{144}$$
$$= 6 \cdot 12$$
$$= 72$$

Simplify. Assume that all variables represent positive real numbers.

1. $6\sqrt{2} \cdot 3\sqrt{2}$ **36**
2. $3\sqrt{5} \cdot 2\sqrt{5}$ **30**
3. $\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{9}$ **6**
4. $\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{16}$ **12**
5. $2\sqrt{3} \cdot \sqrt{5}$ **$2\sqrt{15}$**
6. $4\sqrt{2} \cdot \sqrt{3}$ **$4\sqrt{6}$**
7. $\sqrt{2} \cdot \sqrt{32}$ **8**
8. $\sqrt{3} \cdot \sqrt{27}$ **9**
9. $\sqrt{11} \cdot \sqrt{99}$ **33**
10. $\sqrt{8} \cdot \sqrt{18}$ **12**
11. $4\sqrt{108}$ **$24\sqrt{3}$**
12. $7\sqrt{80}$ **$28\sqrt{5}$**

Example 2 Simplify $\sqrt{\dfrac{7}{6}} \cdot \sqrt{\dfrac{54}{28}}$.

Solution
$$\sqrt{\frac{7}{6}} \cdot \sqrt{\frac{54}{28}} = \sqrt{\frac{7}{6} \cdot \frac{54}{28}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Simplify. Assume that all variables represent positive real numbers.

13. $\sqrt{\dfrac{7}{10}} \cdot \sqrt{\dfrac{10}{7}}$ **1**
14. $\sqrt{\dfrac{5}{3}} \cdot \sqrt{\dfrac{3}{20}}$ **$\dfrac{1}{2}$**
15. $\sqrt{\dfrac{24}{11}} \cdot \sqrt{\dfrac{33}{2}}$ **6**
16. $\sqrt{\dfrac{7}{5}} \cdot \sqrt{\dfrac{5}{28}}$ **$\dfrac{1}{2}$**

17. $\sqrt{\dfrac{3}{8}} \cdot \sqrt{\dfrac{8}{27}}$ **$\dfrac{1}{3}$**
18. $\sqrt{\dfrac{3}{5}} \cdot \sqrt{\dfrac{125}{3}}$ **5**
19. $\sqrt{\dfrac{7}{3}} \cdot \sqrt{\dfrac{3}{112}}$ **$\dfrac{1}{4}$**
20. $\sqrt{\dfrac{2}{5}} \cdot \sqrt{\dfrac{10}{8}}$ **$\sqrt{\dfrac{1}{2}}$, or $\dfrac{\sqrt{2}}{2}$**

11-7 Multiplying, Dividing, and Simplifying Radicals (continued)

Example 3 Simplify: **a.** $\dfrac{2}{\sqrt{3}}$ **b.** $\sqrt{\dfrac{5}{8}}$ **c.** $\dfrac{5\sqrt{2}}{\sqrt{12}}$ **d.** $\sqrt{4\dfrac{4}{5}} \cdot \sqrt{3\dfrac{1}{3}}$

Solution
a. $\dfrac{2}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{\left(\sqrt{3}\right)^2} = \dfrac{2\sqrt{3}}{3}$

b. $\sqrt{\dfrac{5}{8}} = \dfrac{\sqrt{5}}{\sqrt{8}} = \dfrac{\sqrt{5}}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{5 \cdot 2}}{2\left(\sqrt{2}\right)^2} = \dfrac{\sqrt{10}}{4}$

c. $\dfrac{5\sqrt{2}}{\sqrt{12}} = \dfrac{5\sqrt{2}}{\sqrt{2^2 \cdot 3}} = \dfrac{5\sqrt{2}}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{5\sqrt{6}}{2\left(\sqrt{3}\right)^2} = \dfrac{5\sqrt{6}}{6}$

d. $\sqrt{4\dfrac{4}{5}} \cdot \sqrt{3\dfrac{1}{3}} = \sqrt{\dfrac{24}{5}} \cdot \sqrt{\dfrac{10}{3}} = \sqrt{\dfrac{24}{5} \cdot \dfrac{10}{3}} = \sqrt{16} = 4$

Simplify.

21. $\dfrac{3}{\sqrt{5}}$ **$\dfrac{3\sqrt{5}}{5}$**
22. $\dfrac{4}{\sqrt{6}}$ **$\dfrac{2\sqrt{6}}{3}$**
23. $\sqrt{\dfrac{1}{6}}$ **$\dfrac{\sqrt{6}}{6}$**
24. $\sqrt{\dfrac{3}{8}}$ **$\dfrac{\sqrt{6}}{4}$**

25. $\dfrac{6\sqrt{5}}{\sqrt{80}}$ **$\dfrac{3}{2}$**
26. $\dfrac{2\sqrt{3}}{\sqrt{48}}$ **$\dfrac{1}{2}$**
27. $\sqrt{3\dfrac{3}{4}} \cdot \sqrt{2\dfrac{2}{3}}$ **$\sqrt{10}$**
28. $\sqrt{1\dfrac{1}{6}} \cdot \sqrt{4\dfrac{2}{3}}$ **$\dfrac{7}{3}$**

Example 4 Simplify $\sqrt{3}(\sqrt{3} - 4)$.

Solution
$$\sqrt{3}(\sqrt{3} - 4) = \sqrt{3} \cdot \sqrt{3} - \sqrt{3} \cdot 4$$
$$= 3 - 4\sqrt{3}$$

Simplify.

29. $\sqrt{2}(\sqrt{2} - 1)$ **$2 - \sqrt{2}$**
30. $\sqrt{6}(5 - \sqrt{6})$ **$5\sqrt{6} - 6$**
31. $2\sqrt{3}(\sqrt{27} - \sqrt{3})$ **12**
32. $3\sqrt{5}(2\sqrt{5} - \sqrt{125})$ **−45**

Mixed Review Exercises

Solve.

1. $x^2 = 121$ **{−11, 11}**
2. $2x^2 - 128 = 0$ **{−8, 8}**
3. $16x^2 - 1 = 24$ **$\left\{-\dfrac{5}{4}, \dfrac{5}{4}\right\}$**
4. $\dfrac{1}{c} + \dfrac{1}{3} = \dfrac{1}{2}$ **{6}**
5. $\dfrac{3}{5} = \dfrac{15}{y}$ **{25}**
6. $\dfrac{6b - 1}{3b - 1} = \dfrac{5}{2}$ **{1}**

Simplify.
7. $15x + 2(3x - 5) + 2$ **$21x - 8$**
8. $10a + 6 - (6a - 12)$ **$4a + 18$**
9. $3(2a - 5) - 4(a - 3)$ **$2a - 3$**
10. $(-4cd^2)(-3c^2d)$ **$12c^3d^3$**
11. $-3m + 2 + 9m - 5$ **$6m - 3$**
12. $x(x - 1) + (x - 3)(2x - 1)$ **$3x^2 - 8x + 3$**